# N2 • Evaluating statements about number operations

Mathematical goals

To enable learners to:

- understand the properties of number operations;
- substitute integers, fractions, decimals and negative numbers into statements and equations in order to test their validity;
- address common misconceptions about the effect of addition, subtraction, multiplication, division, squaring and finding square roots.
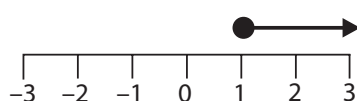
Some learners may also begin to work with number line notation such as the example (left) representing $x \geq 1$.

Starting points

Although learners will be familiar with number operations, many will still have problems interpreting and using these concepts. This session is designed to expose and work on difficulties that learners have with:

- adding and subtracting negative numbers;
- interpreting division notation (e.g. misreading $x \div y$ as "how many xs go into y?");
- the commutativity of multiplication but not of division;
- the effect of dividing a smaller number by a larger number;
- the effect of multiplying and dividing by numbers less than 1.

Materials required

For each learner you will need:

- Sheet 1 – *Number operations*.

For each small group of learners you will need:

- Card set A – *Statements*;
- Card set B – *Number lines*;
- large sheet of paper for making a poster;
- glue stick;
- felt tip pen.

Calculators may be helpful for learners to check their answers.

Time needed

At least 1 hour. The exact time depends on whether or not you choose to use Card set B – *Number lines*.

Suggested approach

Beginning the session

As learners arrive, ask them to complete Sheet 1 – *Number operations*, working on their own. Learners frequently comment that it looks easy, and complete it in just a few minutes. It should, however, expose many misconceptions that can be discussed later in the session. Do not go through the answers at this stage. Tell learners to put the sheet to one side, face down, for the time being.

Write one of the harder statements from Card set A – *Statements* on the board e.g. $x^2 \geq x$. Ask learners to interpret this statement in words and to say whether they think it is a true statement or not. Typically, learners begin by saying that this is clearly true because, when you multiply a number by itself, it gets bigger. Ask questions about the statement:

> Can you give me a value for x that makes the statement true?
> Can you give me another? And another?
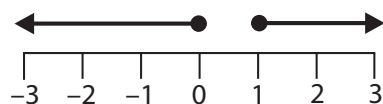> Try a fraction, a decimal, a negative number . . .
>
> Can you give me a value for x that makes the statement false?
> Can you give me another? And another?
> Try a fraction, a decimal, a negative number . . .
>
> Can we state precisely when the statement is true and when it is not?

Demonstrate how the result may be shown on a number line:

Explain the notation, where solid blobs denote that you do include the number and hollow blobs denote that you do not.

Explain that in this session learners will be asked to consider a number of statements in a similar way. Explain that the objective of the session is for each small group of learners to produce a poster which shows each statement classified according to whether it is always, sometimes or never true and furthermore:

- if it is sometimes true, then to write examples around the statement to show when it is true and when it is not true;

- if it is always true, then to give a variety of examples demonstrating that it is true, using large numbers, decimals, fractions and negative numbers if possible;

- if it is never true, then to write an explanation of how you can be sure that this is the case.

Working in groups

Ask learners to work in groups of two or three.

Give each group Card set A – *Statements*, a large sheet of paper, a glue stick and a felt tip pen.

Ask learners to divide their sheet into three columns and to head the columns with the words: 'Always True', 'Sometimes True', 'Never True'.

Learners now take it in turns to place a card in one of the columns and justify their answer to their partner(s). Their partner(s) must challenge them if the explanation has not been clear and complete. When the pair or group agrees, they should paste the card down and write examples around it to justify their choice. This should include examples and counter-examples. Learners should not need to rearrange the equations. Trial-and-error substitutions should be enough in most cases.

Learners who struggle should be given calculators to help with the arithmetic. Suggesting numbers for learners to substitute may help to take their thinking forward.

Learners who need an extra challenge should be encouraged to match the *Number lines* cards (Card set B) to the *Statements* cards (Card set A). For even greater challenge, you may wish to add further, more demanding, cards such as $x^2 + 4 = 13$.

When doing this activity, you may find that learners sort their cards quickly and superficially to begin with. They may need prompting to try fraction, decimal and negative substitutions to check their assumptions, if they do not do this of their own accord. Look for common misconceptions that surface and note these down for later discussion with the whole group.

Reviewing and extending learning

Ask learners to display their posters to the whole group and to describe one thing they have learned.

Name particular misconceptions you have identified as learners were working on the activity. For example:

- division is commutative ($10 \div x = x \div 10$);
- you cannot divide smaller numbers by larger ones;
- addition/multiplication/squaring always makes numbers bigger;
- subtraction/division/square rooting always makes numbers smaller;

. . . and so on.

Finally, ask learners to look again at the questions they attempted at the beginning of the session. They should correct any that they now know are incorrect and write down what they have learned in the space underneath.

**What learners
might do next**

A logical follow-up to this session is **N5 Understanding the laws of arithmetic**, which focuses on the order in which number operations are carried out.

Further ideas

This activity is about examining a mathematical statement and deciding on its truth or falsehood. This idea may be used in many other topics and levels. Examples in this pack include:

A4   **Evaluating algebraic expressions;**

SS4   **Evaluating statements about length and area;**

S2   **Evaluating probability statements.**

N2 Sheet 1 – *Number operations*

Write down the missing numbers in the following.

You will have another go at this sheet at the end of the session. This will help you to identify what you have learned.

1. $12 + 6 = \ldots$

   $6 + 12 = \ldots$

5. $12 + \ldots = 24$

   $12 - \ldots = 24$

2. $12 - 6 = \ldots$

   $6 - 12 = \ldots$

6. $12 + \ldots = 6$

   $12 - \ldots = 6$

3. $12 \times 6 = \ldots$

   $6 \times 12 = \ldots$

7. $12 \times \ldots = 24$

   $12 \div \ldots = 24$

4. $12 \div 6 = \ldots$

   $6 \div 12 = \ldots$

8. $12 \times \ldots = 6$

   $12 \div \ldots = 6$
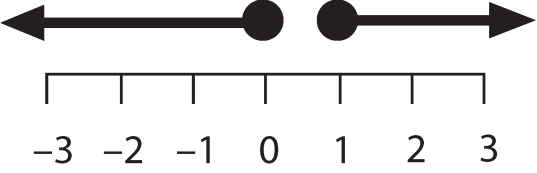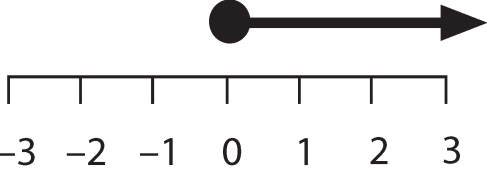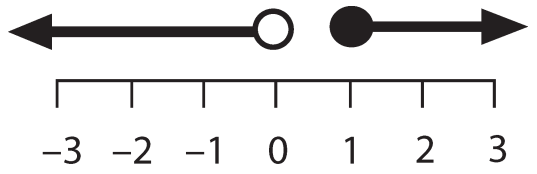
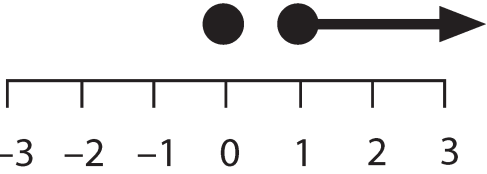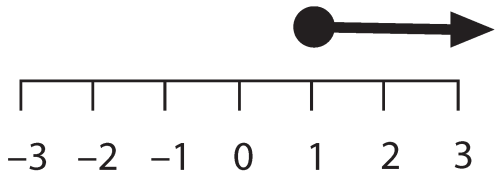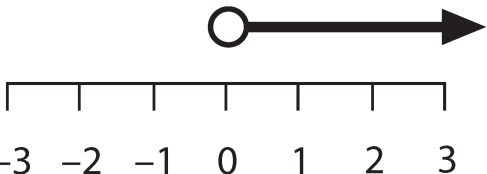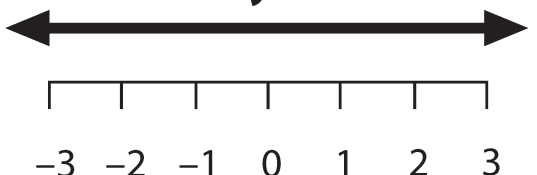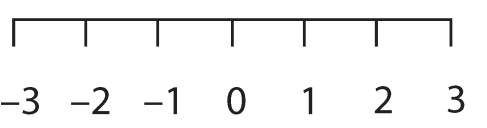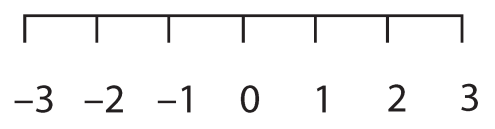At the end of the session, write down what you have learned in the space below.

If you still have questions or difficulties, write these down too.

$3 + x = x + 3$ **It doesn't matter which way round you add, you get the same answer.**	$2 - x = x - 2$ **It doesn't matter which way round you subtract, you get the same answer.**
$5 \times x = x \times 5$ **It doesn't matter which way round you multiply, you get the same answer.**	$x \div 2 = 2 \div x$ **It doesn't matter which way round you divide, you get the same answer.**
$5 + x > 5$ **If you add a number to 5, your answer will be more than 5.**	$x + 8 > x$ **If you add 8 to a number, your answer will be more than the number.**
$5 - x \leq 5$ **If you take a number away from 5, your answer will be less than or equal to 5.**	$x - 10 > x$ **If you take 10 away from a number, the answer will be greater than the number.**
$4x \geq 4$ **If you multiply 4 by a number, your answer will be greater than or equal to 4.**	$10x \geq x$ **If you multiply 10 by a number, your answer will be greater than or equal to the number.**
$\dfrac{x}{2} < x$ **If you divide a number by 2, the answer will be less than the number.**	$\dfrac{10}{x} \leq 10$ **If you divide 10 by a number, your answer will be less than or equal to 10.**
$\sqrt{x} \leq x$ **The square root of a number is less than or equal to the number.**	$x^2 \geq x$ **The square of a number is greater than or equal to the number.**

B1 **Sometimes true** ◀━━━━━●━━●━━━▶ −3 −2 −1 0 1 2 3	**B2** **Sometimes true** ●━━━━━━━━▶ −3 −2 −1 0 1 2 3
B3 **Sometimes true** ◀━━━━━○━━●━━━▶ −3 −2 −1 0 1 2 3	**B4** **Sometimes true** ●━━●━━━▶ −3 −2 −1 0 1 2 3
B5 **Sometimes true** ●━━━▶ −3 −2 −1 0 1 2 3	**B6** **Sometimes true** ○━━━━━▶ −3 −2 −1 0 1 2 3
B7 **Always true** ◀━━━━━━━━━━━▶ −3 −2 −1 0 1 2 3	**B8** **Never true** −3 −2 −1 0 1 2 3
B9 −3 −2 −1 0 1 2 3	**B10** −3 −2 −1 0 1 2 3